2020 STEP CONFERENCE

INTRODUCTION TO CIRCUITRY AND POWER

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Chapter 1

Lecture

1.1 INTRODUCTION

This module is designed to introduce students to the basic concepts of circuitry and power.

Objectives

- Understand Ohm's Law
- Understand Series and Parallel Circuits
- Understand how to read a basic circuit schematic
- Understands the basics phyics of power and energy
- Apply the concepts mentioned above to a circuit simulation
- Understand how to solve basic circuits mathematically
- Be introduced to a university-level engineering problem (Thévenin's Theorem)
- Have fun

1.2 Ohm's Law

Ohm's Law describes the relationship between voltage, current, and resistance.

1.2.1 Resistance

This is described as a division between voltage and current.

$$R = \frac{V}{I} \tag{1.1}$$

The unit of measurement is usually described in Ohms (Ω)

1.2.2 Current

This is described as a division between voltage and resistance.

$$I = \frac{V}{R} \tag{1.2}$$

The unit of measurement is usually described in Ampère, Amps, A*

*All are the same unit just different symbols

1.2.3 Voltage

This is described as a multiplication of resistance and current.

$$V = R \times I \tag{1.3}$$

The unit of measurement is usually described in Volts (V)

1.3 Series Circuit

A circuit is in series when two or more loads only share one node in a consecutive matter.

- Loads may have different voltages
- Loads have the same value for current
- Loads may consume different amounts of power

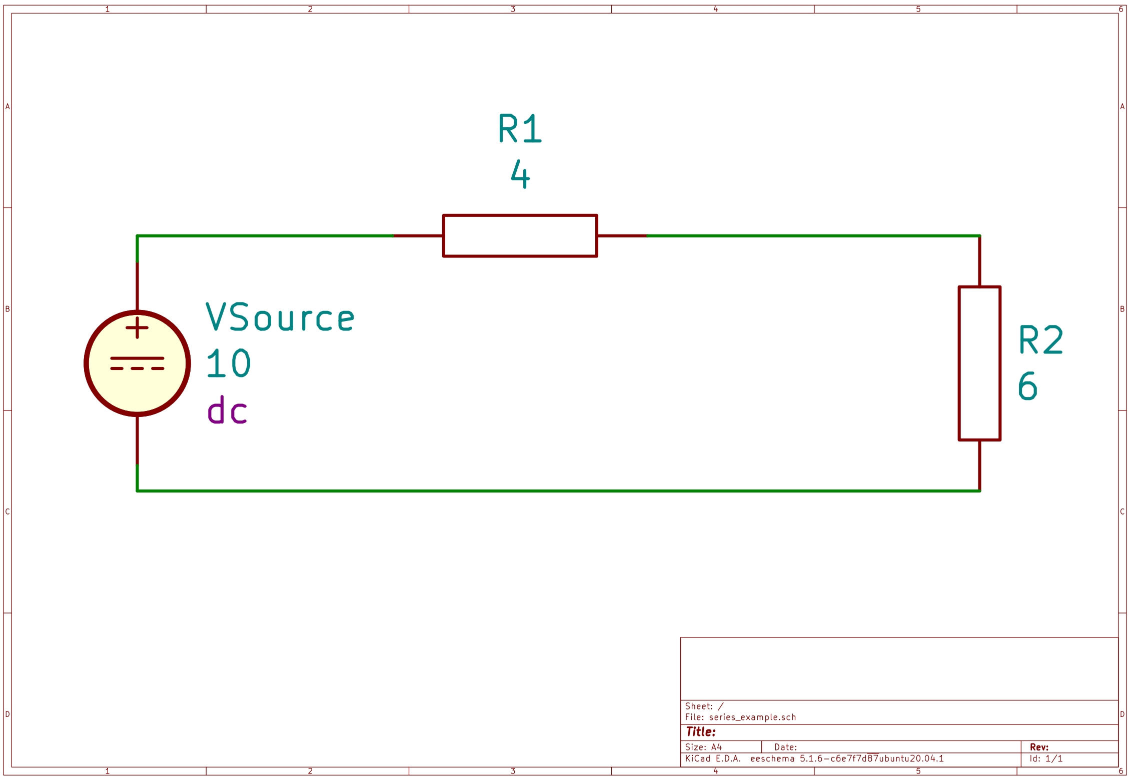

Figure 1.1: Example of a Series Circuit

1.3.1 Equivalent Resistance

Equivalent Resistance represents all of the resistors as one resistor. In series circuits, this is the summation of all the resistors as shown in equation 4.

$$R_{eq} = \sum_{i=1}^{n} R_i \tag{1.4}$$

1.3.2 Example

Using figure 1.1, one can calculate the equivalent resistance using equation 1.4.

$$R_{eq} = R_1 + R_2$$

$$R_{eq} = 4\,\Omega + 6\,\Omega = 10\,\Omega$$

1.4 Parallel Circuit

A circuit is in parallel when two or more loads share both nodes.

- Loads will have the same voltage
- Loads may have different values for currents
- Loads may consume different amounts of power

1.4.1 Equivalent Resistance

In parallel circuits, the equivalent resistance is found by taking the inverse of all resistors, then summing all of those inverses together, and lastly taking the inverse of that sum. This is shown in equation 1.5.

$$R_{eq} = \left(\sum_{i=1}^{n} {R_i}^{-1}\right)^{-1} \tag{1.5}$$

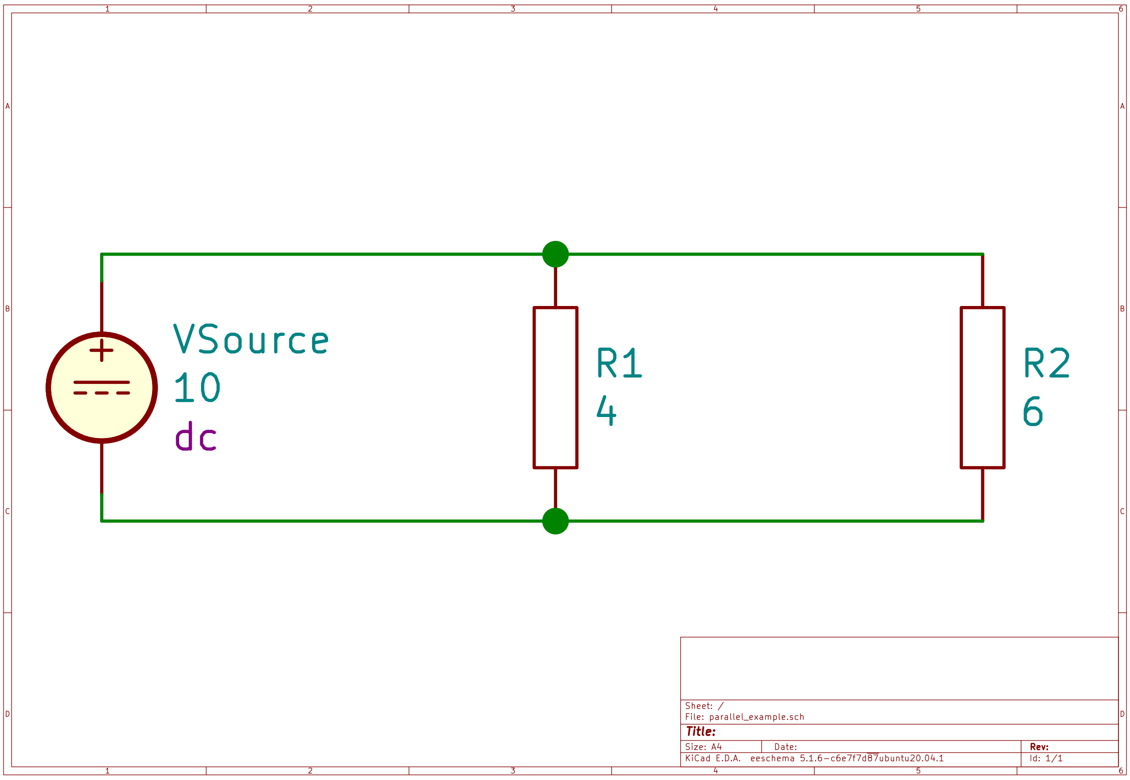

Figure 1.2: Example of a Parallel Circuit

1.4.1.1 Shortcut

In the event you ONLY have TWO loads, you may use this equation

$$R_{eq} = \frac{R_1 \times R_2}{R_1 + R_2} \tag{1.6}$$

1.4.2 Example

Using figure 1.2, one can calculate the equivalent resistance using equation 1.5.

$$R_{eq} = \left(R_1^{-1} + R_2^{-1}\right)^{-1} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} = \frac{R_1 \times R_2}{R_1 + R_2}$$

$$R_{eq} = \left(4\,\Omega^{-1} + 6\,\Omega^{-1}\right)^{-1} = \frac{1}{\frac{1}{4\,\Omega} + \frac{1}{6\,\Omega}} = \frac{6\,\Omega \times 4\,\Omega}{6\,\Omega + 4\,\Omega} = 2.4\,\Omega$$

1.5 Power

Power is the amount of energy consumed during an certain amount of time. Watt (W) is the unit of measurement for power.

$$W = \frac{J}{s} = \frac{N \times m}{s} \tag{1.7}$$

That is the mechanical definition for power, however the electrical definition is critical for circuit analyis.

$$W = V \times I \tag{1.8}$$

$$W = \frac{V^2}{R} \tag{1.9}$$

$$W = I^2 \times R \tag{1.10}$$

$$W = V \times I = \frac{V^2}{R} = I^2 \times R \tag{1.11}$$

1.5.1 Calculating the amount of power in circuits

A simple circuit can be solved using equation 1.8.

1.5.1.1 Series Circuits

In series circuits, all the loads share the same current. In this scenario, equation 1.10 is optimal since the current needs to be calculated once.

1.5.1.2 Example

Using figure 1.1, one can calculate the power consumed for each resistor using equation 1.10.

$$Current\,(I) = 1\ A$$
$$W = I^2 \times R$$
$$P_{R_1} = I^2 \times R_1$$
$$P_{R_1} = 1\ A \times 4\ \Omega = 4\ W$$
$$P_{R_2} = I^2 \times R_2$$
$$P_{R_2} = 1\ A \times 6\ \Omega = 6\ W$$

1.5.1.3 Parallel Circuits

In parallel circuits, the voltages for all the resistors are equal. In this scenario, equation 9 is optimal since the voltage needs to be calculated once.

1.5.1.4 Example

Using figure 1.2, one can calculate the power consumed for each resistor using equation 1.9.

$$Voltage\,(V) = 10\ V$$
$$W = \frac{V^2}{R}$$
$$P_{R_1} = \frac{V^2}{R_1}$$
$$P_{R_1} = \frac{100\ V}{4\ \Omega} = 25\ W$$
$$P_{R_2} = \frac{V^2}{R_2}$$
$$P_{R_2} = \frac{100\ V}{6\ \Omega} = 16.\overline{6}\ W$$

1.6 Divider Circuits

When analyzing circuits, it is important to know what the voltage and current will be at certain points. The voltage and current divider are basic tools for determing voltage and current.

1.6.1 Voltage Divider

When the resistors are in series, use equation 1.12 to find the voltage bewtween the resistors.

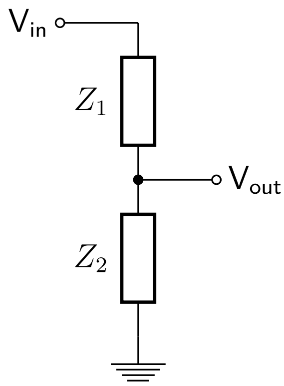

Figure 1.3: Example of a Voltage Divider Circuit (Z is the same as R)

$$V_{out} = V_{in}\left(\frac{R_{afterpoint}}{R_{beforepoint}+R_{afterpoint}}\right)^{*} \tag{1.12}$$

*$R_{beforepoint}$ is/are the resistor(s) before the midpoint(V_{out}), and $R_{afterpoint}$ is/are the resistor(s) after the midpoint.

1.6.1.1 Example

Find the voltage inbetween Resistor 1 and 2 in figure 1.1.

$$R_{beforepoint} = R_1$$

$$R_{afterpoint} = R_2$$

$$V_{in} = 10\ V$$

$$V_{out} = V_{in}\left(\frac{R_{afterpoint}}{R_{beforepoint} + R_{afterpoint}}\right)$$

$$V_{out} = V_{in}\left(\frac{R_2}{R_1 + R_2}\right)$$

$$V_{out} = 10\ V\left(\frac{6\ \Omega}{4\ \Omega + 6\ \Omega}\right) = 6\ V$$

1.6.2 Current Divider

When the resistors are in parallel, use equation 1.13 to find the current of each resistor individually.

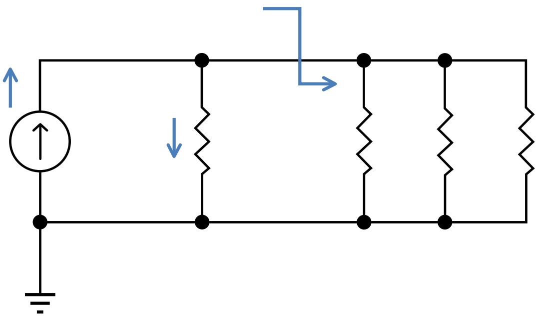

Figure 1.4: Example of a Current Divider Circuit

$$I_{R_x} = I_{in}\left(\frac{R_{other}}{R_x + R_{other}}\right)^* \tag{1.13}$$

* R_x is the resistor one is finding the current for. R_{other} is all the other resistor(s) R_x is in parallel with.

1.6.2.1 Example

Find the current in Resistor 1 in figure 2.

$$I_{in} = 2.4\ A$$

$$R_x = R_1$$

$$R_{other} = R_2$$

$$I_{R_x} = I_{in}\left(\frac{R_{other}}{R_x + R_{other}}\right)$$

$$I_{R_1} = I_{in}\left(\frac{R_2}{R_1 + R_2}\right)$$

$$I_{R_1} = 2.4\ V\left(\frac{6\,\Omega}{4\,\Omega + 6}\right) = 1.44\ A$$

1.7 THÉVENIN'S THEOREM

Understanding parallel and series circuits is the basis for circuit analysis, however circuits can get more complicated than series and parallel circuits. Understanding the different methods for complex circuit analysis is one of main goals of every undergraduate electrical engineering student. This mini-lecture will give a snapshot of those sophisticated circuit analysis methods. This is a challenge and not something expected for high school level, just try your best and have fun with it. :)

1.7.1 Basic Theory

If there is a complicated circuit, but only one resistor is of interest, the entire circuit can be abtstracted to one voltage source, and one resitor for the rest of the circuit excluding the resistor of interest.

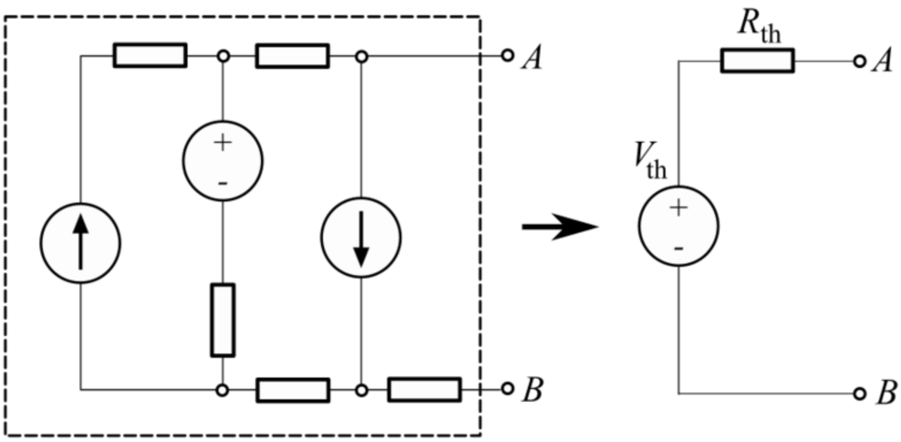

Figure 1.5: Thévenin's Theorem

1.7.1.1 Finding $V_{Thévenin}$

1. Disconnect the resistor from the circuit

2. Find the voltage inbetween the two open nodes (connections) of the circuit

3. This gives you $V_{Thévenin}$

1.7.1.2 Finding $R_{Thévenin}$

1. Disconnect the resistor from the circuit
2. Replace the voltage source(s) with wire(s)
3. Find the R_{eq} of this cicuit
4. This gives you $R_{Thévenin}$

1.7.2 Example

Find $V_{Thévenin}$ and $R_{Thévenin}$ in figure 1.6.

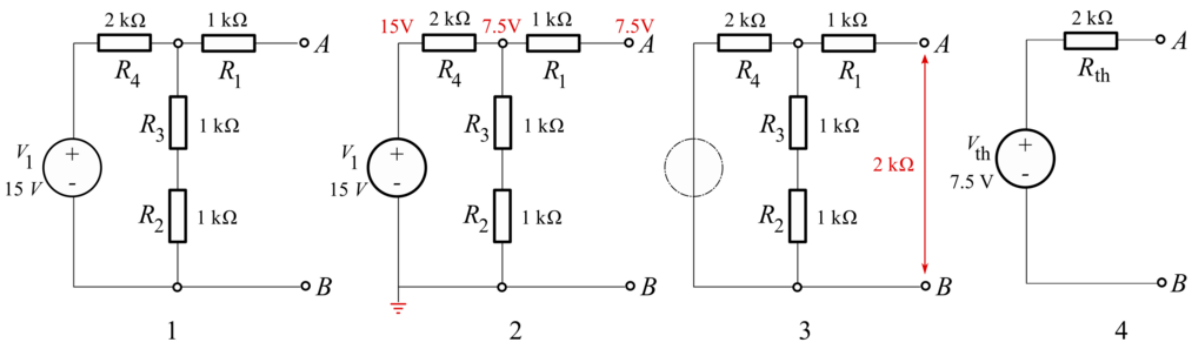

Figure 1.6: Thévenin's Theorem example

1. Original circuit
2. The equivalent voltage
3. The equivalent resistance
4. The equivalent circuit

1.7.2.1 Find $V_{Thévenin}$

$$
\begin{aligned}
V_{\text{Th}} &= \frac{R_2 + R_3}{(R_2 + R_3) + R_4} \cdot V_1 \\
&= \frac{1\,\text{k}\Omega + 1\,\text{k}\Omega}{(1\,\text{k}\Omega + 1\,\text{k}\Omega) + 2\,\text{k}\Omega} \cdot 15\,\text{V} \\
&= \frac{1}{2} \cdot 15\,\text{V} = 7.5\,\text{V}
\end{aligned}
$$

1.7.2.2 Find $R_{Thévenin}$

$$
R_{\text{Th}} = R_1 + [(R_2 + R_3) \parallel R_4]
$$

$$
= 1\,\text{k}\Omega + [(1\,\text{k}\Omega + 1\,\text{k}\Omega) \parallel 2\,\text{k}\Omega]
$$

$$
= 1\,\text{k}\Omega + \left(\frac{1}{(1\,\text{k}\Omega + 1\,\text{k}\Omega)} + \frac{1}{(2\,\text{k}\Omega)}\right)^{-1} = 2\,\text{k}\Omega.
$$

Chapter 2

Assignment

PhET simulations are interactive simulations used for STEM eductation. There are a plethora of simulations ranging from biology, chemistry, physics, and math. I highly reccomend checking the other simulations PhET has to offer at this link:
https://phet.colorado.edu/
For the purposes of this assignment, only the Circuit Construction Kit: DC - Virtual Lab is needed. Click on this link to access the lab:
https://phet.colorado.edu/sims/html/circuit-construction-kit-dc-virtual-lab/latest/circuit-construction-kit-dc-virtual-lab_en.html

2.1 TOOLS

The DC circuit has 3 components for our needs: wires, batteries, and resistors. While this may seem like a rudimentaty setup, power systems has its foundations buit upon simple components. Wires in the power systems do have a resitance, but in this exercise; the resistance is negligible. The battery is a power source which can include genrators, voltage power supplies, current power supplies etc. The resistor represents a load in power systems, while a resitor may seem like a trivial component, all componnets that consume power are treated as loads. An electric company does not care what light bulb or electric vehicle is used, the only attribute that matters is how much the load consumes. In summary, the goal is to understand basic power sytems component which are: wires, power sources, and loads.

2.1.1 Measuring Voltage

Once the circuit is created, voltage needs to be measured. Using the two voltmeter(measures voltage) ends: one positive and negative, measure across the given voltage potential. Switching the tips will give you the same value, but the postive/neagtive sign is switched. It should be noted there will be two values for measuring voltage V_x and V_{xn}. V_x is the voltage across the resistor, while V_{xn} is from the resistor node to the negative end of the battery. V_x represents the voltage drop across the resistor, while V_{xn} is the voltage relative to the system. The system uses the negative end of the battery as its point of reference.

2.1.2 Measuring Current

Measuring current is more diffcullt since you need to "break the circuit". To measure the current of a component, disconncet one node of the componet, and put the ammeter (measures current) in series with the component. Current measurments do not need a reference point, so only one measurement is needed for each resistor. To measure the total current of the circuit, put the ammeter on either side of the voltage source before any other components.

2.2 PROBLEMS

Solve the circuits mathematically by hand or code, and verify with the circuit simulation. Fill out the spreadsheet with the results calculated.

2.2.1 Problem 1

Use the series, power, and/or divider equations to solve the circuit.

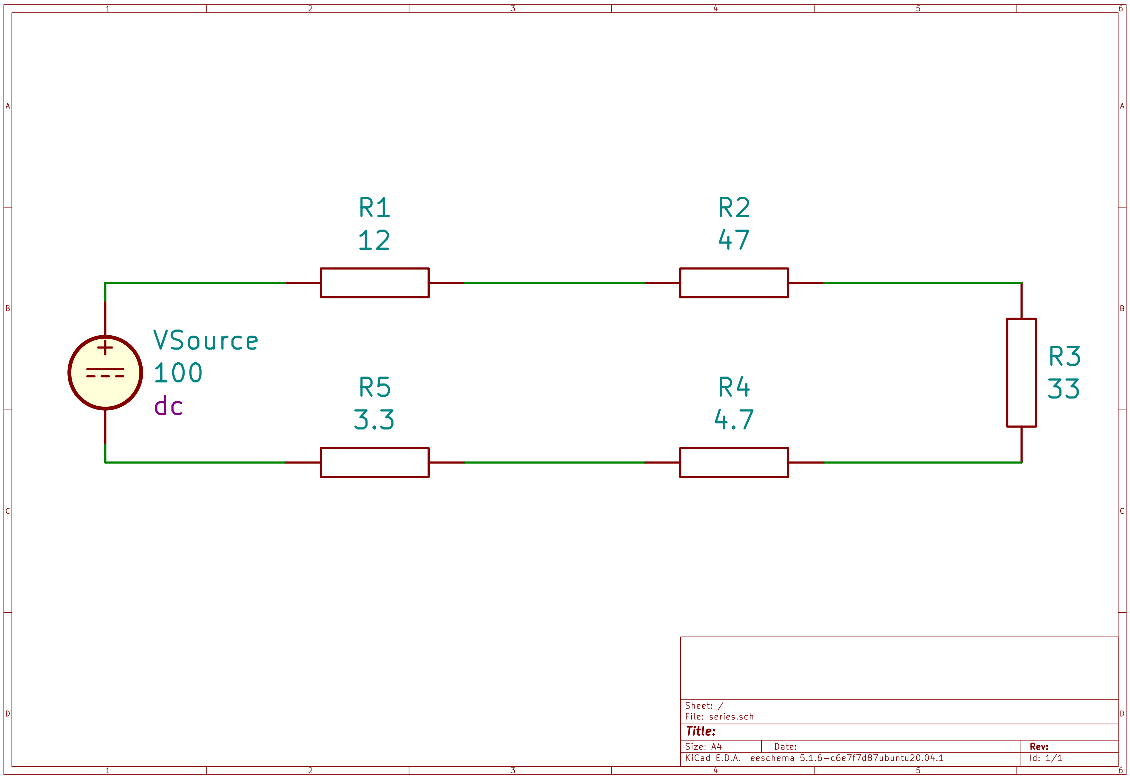

Figure 2.1: Circuit for Problem 1

2.2.2 Problem 2

Use the parallel, power, and/or divider equations to solve the circuit.

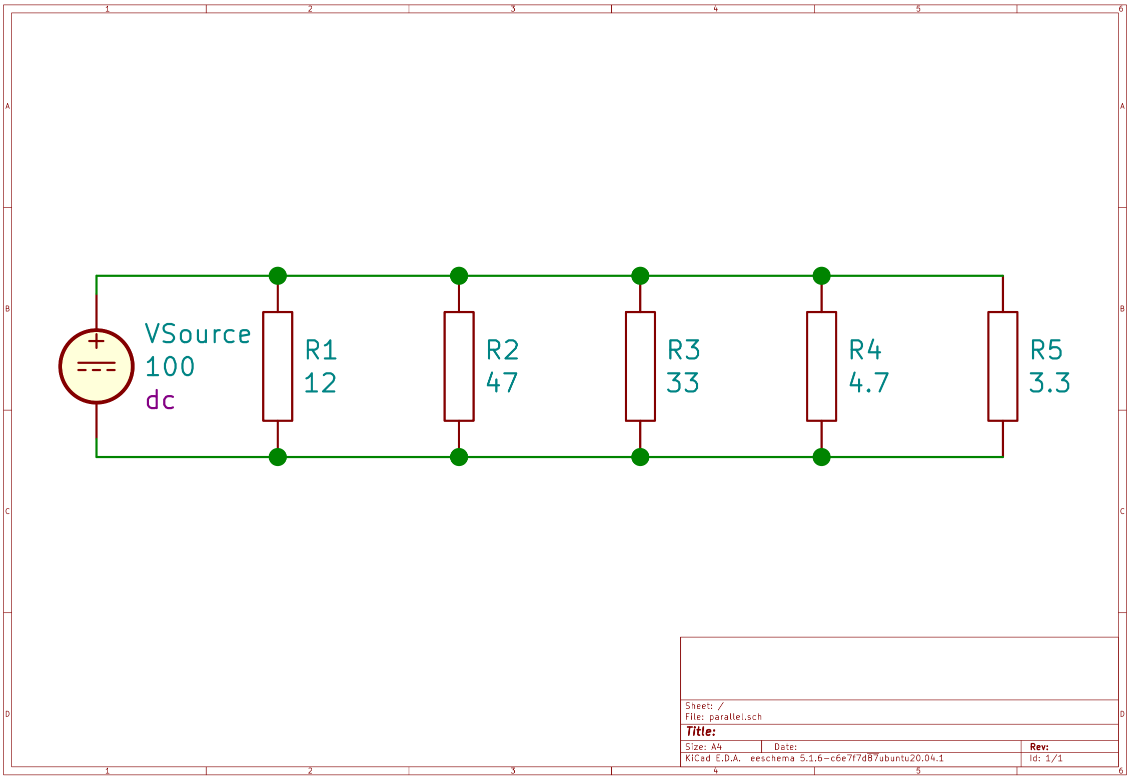

Figure 2.2: Circuit for Problem 2

2.2.3 Problem 3

Use the series, parallel, power, and/or divider equations to solve the circuit.

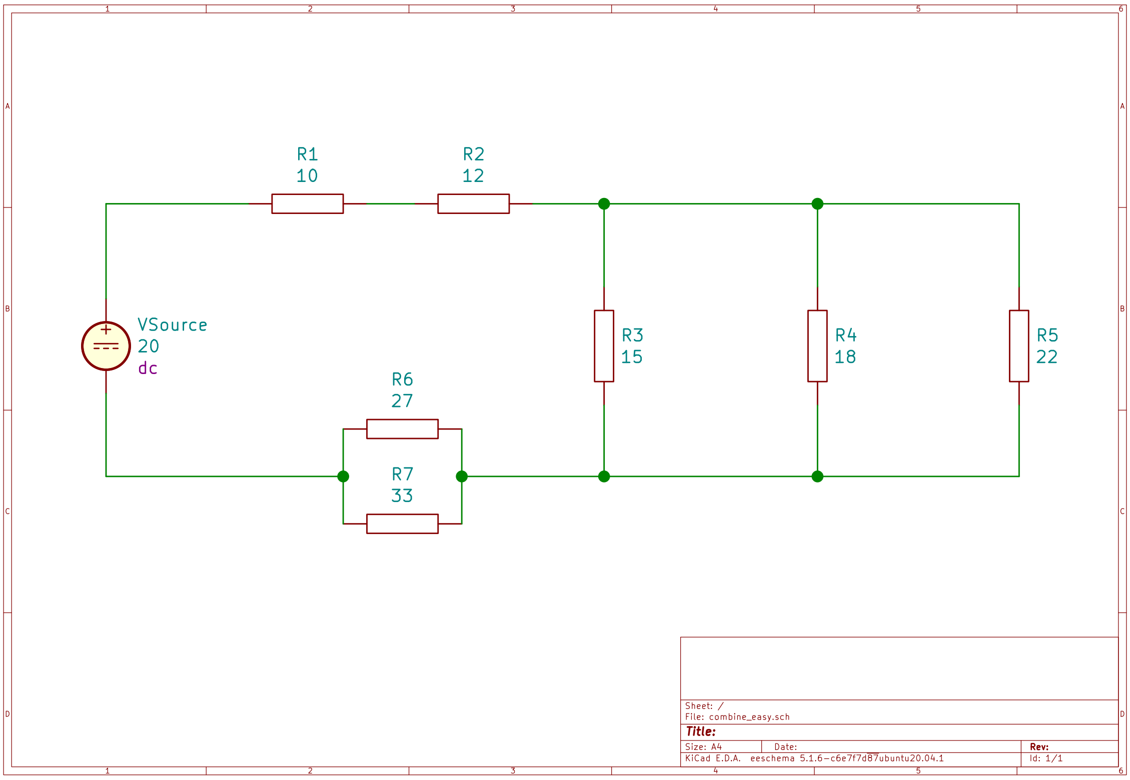

Figure 2.3: Circuit for Problem 3

2.2.4 Problem 4

Use the series, parallel, power, and/or divider equations to solve the circuit.

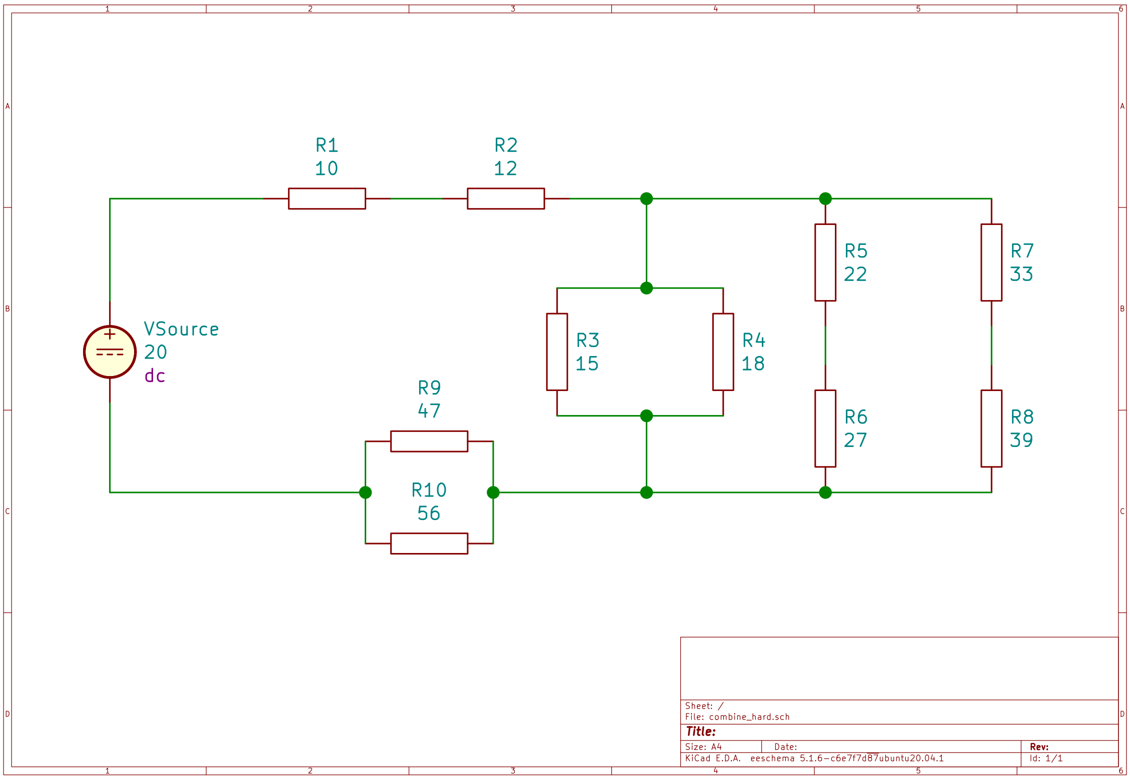

Figure 2.4: Circuit for Problem 4

2.3 CHALLENGE ASSIGNMENT

Use Thévenin's Theorem to find the power consumed by R_x.

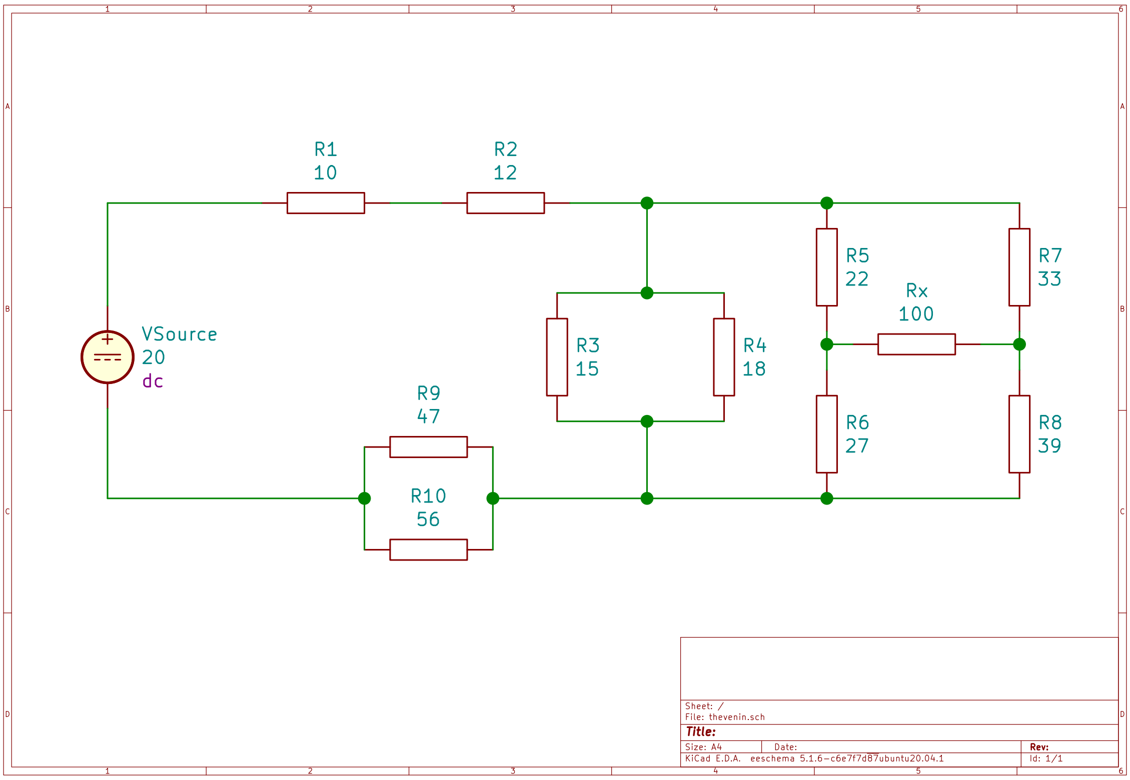

Figure 2.5: Circuit for Challenge Problem

Bibliography

@misc wiki:xxx, author = "Wikipedia contributors", title = "Voltage divider — Wikipedia, The Free Encyclopedia", year = "2020", howpublished = "https://en.wikipedia.org/w/index.php?title=Voltage_divider&oldid=946933383", note = "[Online; accessed 16-September-2020]" @misc wiki:xxx, author = "Wikipedia contributors", title = "Current divider — Wikipedia, The Free Encyclopedia", year = "2020", howpublished = "https://en.wikipedia.org/w/index.php?title=Current_divider&oldid=977713039", note = "[Online; accessed 16-September-2020]" @misc wiki:xxx, author = "Wikipedia contributors", title = "Thévenin's theorem — Wikipedia, The Free Encyclopedia", year = "2020", howpublished = "https://en.wikipedia.org/w/index.php?title=Th%C3%A9venin%27s_theorem&oldid=976237789", note = "[Online; accessed 16-September-2020]"